



Class 2: Number Sense

Warm-up Problem 1

Mental Math – Calculate the following without using a calculator or pencil/paper as quick as possible.

$$\begin{aligned} \text{(a)} \quad & 202 - 101 + 9 \\ & = 110 \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad & 2 \times 9 + 2 \\ & = 20 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & 245 + 6 - 45 \\ & = 206 \end{aligned}$$

$$\begin{aligned} \text{(f)} \quad & 5 \times 1 - 1 \\ & = 4 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & 483 - 13 + 26 \\ & = 496 \end{aligned}$$

$$\begin{aligned} \text{(g)} \quad & 28 + 32 \\ & = 60 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad & 199 + 31 \\ & = 230 \end{aligned}$$

$$\begin{aligned} \text{(h)} \quad & 5 \times 2 \times 3 + 2 \\ & = 32 \end{aligned}$$

BEDMAS

When doing some operations, we have to follow a specific order to do the operations in.

For example, with the expression, $1 + 5 \times 6$, we might find two ways to do it:

A. $1 + 5 \times 6 = 6 \times 6 = 36$ (incorrect answer)

B. $1 + 5 \times 6 = 1 + 30 = 31$ (correct answer)

We use BEDMAS to determine which order we calculate the operations in.

B = Brackets

E = Exponents

D = Division

M = Multiplication

A = Addition

S = Subtraction

} Order can be switched between these

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Warm-up Problem 2

Calculate the value of the following:

(a) $5 \times 3 - 2$

(b) $6 + 2 \times 3$

(c) $5 - 6 \div 3$

(d) $8 \div 2(2 + 2)$

(a) $5 \times 3 - 2$

$= 15 - 2$

$= 13$

(b) $6 + 2 \times 3$

$= 6 + 6$

$= 12$

(c) $5 - 6 \div 3$

$= 5 - 2$

$= 3$

(d) $8 \div 2(2 + 2)$

$= 4(2 + 2)$

$= 4 \times 4$

$= 16$

B = Brackets
E = Exponents
D = Division
M = Multiplication
A = Addition
S = Subtraction

Problem (Previous day)

Alex had four math tests this year. His average score on his first three math tests was 86. He worked very hard for the last test and his overall average was improved to 88.

What score did he get on his last test?

$$\text{Total score for first three tests} = 3 \times 86 = 258$$

$$\text{Total score for the whole year} = 4 \times 88 = 352$$

$$\begin{aligned}\text{Score on the last test} &= (\text{Total score for four tests}) - (\text{Total score for first three tests}) \\ &= 352 - 258 \\ &= 94\end{aligned}$$

\therefore Alex scored 94 on his last test.

Problem (Previous day)

The average mass of four people in an elevator was 60 kg. After a fifth person came in the elevator, the average mass became 62 kg. What was the mass of the fifth person?

$$\text{Total mass of first four people} = 4 \times 60 = 240$$

$$\text{Total mass of all five people} = 5 \times 62 = 310$$

$$\begin{aligned}\text{Mass of fifth person} &= (\text{Total mass of first four people}) - (\text{Total mass of all five people}) \\ &= 310 - 240 \\ &= 70\end{aligned}$$

\therefore The mass of the fifth person is 70 kg.

Sequences

A sequence is an ordered list of numbers.

In this class, we will be learning about **arithmetic sequences**, where every two terms in the sequence has the same difference.

Examples of arithmetic sequences:

$1, 2, 3, \dots$

$2, 4, 6, \dots$

$4, 9, 14, 19, \dots$

$15, 10, 5, 0, -5, \dots$

Exercise

Which of the following are arithmetic sequences? Check all that apply.

- A. 1, 4, 7, 11, ...
- B. 5, 6, 7, 8, ...
- C. 10, 25, 40, 65, ...
- D. -1, -5, -9, -13, ...
- E. 5, 9, 13, 17, ...

A. The differences are 3, 3, and 4, so it is NOT an arithmetic sequence.

B. The differences are 1, 1, 1, so it is an arithmetic sequence.

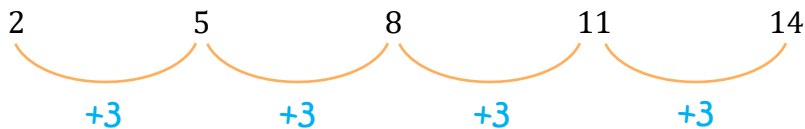
C. The differences are 15, 15, 25, so it is NOT an arithmetic sequence.

D. The differences are -4, -4, -4, so it is an arithmetic sequence.

E. The differences are 4, 4, 4, so it is an arithmetic sequence.

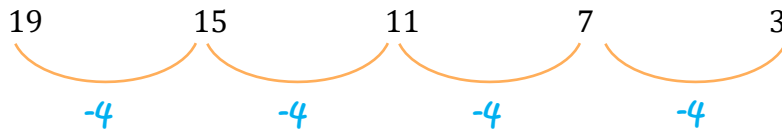
Finding the n^{th} term of a Sequence

(a)



Common Difference = 3

(b)



Common Difference = - 4

n^{th} term = first term + $(n - 1) \times \text{difference}$

$$n^{\text{th}} \text{ term} = \text{first term} + (n - 1) \times \text{difference}$$

Exercise

Find the 8th term of each sequence.

(a) 5, 6, 7, 8, ...

(b) 6, 9, 12, 15, ...

(a) Common difference = 1

First term = 5

$$8^{\text{th}} \text{ term} = 5 + (8 - 1) \times 1$$

$$= 5 + 7$$

$$= 12$$

5, 6, 7, 8, 9, 10, 11, 12

(b) Common difference = 3

First term = 6

$$8^{\text{th}} \text{ term} = 6 + (8 - 1) \times 3$$

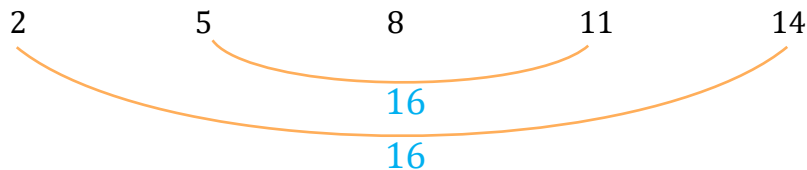
$$= 6 + 7 \times 3$$

$$= 6 + 21$$

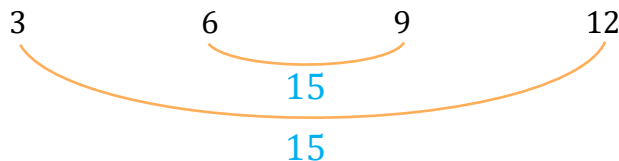
$$= 27$$

6, 9, 12, 15, 18, 21, 24, 27

Sum of a Sequence with Common Difference



$$\begin{aligned}\text{Sum} &= 2 + 5 + 8 + 11 + 14 = 40 \\ \text{Sum} &= 16 \times 5 \div 2 \\ &= 40\end{aligned}$$



$$\begin{aligned}\text{Sum} &= 3 + 6 + 9 + 12 = 30 \\ \text{Sum} &= 15 \times 4 \div 2 \\ &= 30\end{aligned}$$

$$\text{Sum} = (\text{first term} + \text{last term}) \times (\text{number of terms}) \div 2$$

$$n^{\text{th}} \text{ term} = \text{first term} + (n - 1) \times d$$

$$\text{Sum} = (\text{first term} + \text{last term}) \times (\text{number of terms}) \div 2$$

Problem

Given the sequence 5, 12, 19, 26, ...

(a) Find the tenth term of the sequence.

(b) Find the sum of the first ten terms.

$$(a) 10^{\text{th}} \text{ term} = 5 + (10 - 1) \times 7$$

$$= 5 + 9 \times 7$$

$$= 5 + 63$$

$$= 68$$

$$(b) \text{ Sum of first 10 terms} = (5 + 68) \times 10 \div 2$$

$$= 73 \times 10 \div 2$$

$$= 730 \div 2$$

$$= 365$$

$$n^{\text{th}} \text{ term} = \text{first term} + (n - 1) \times d$$

$$\text{Sum} = (\text{first term} + \text{last term}) \times (\text{number of terms}) \div 2$$

Problem

The Olympic Games take place every 4 years. The first Olympic Games was held in 1896. In which year did the 31st Olympics take place?

The years of the Olympic Games form a sequence.

First term = 1896

Common difference = 4

$$\begin{aligned} 31^{\text{st}} \text{ term} &= 1896 + (31 - 1) \times 4 \\ &= 1896 + 30 \times 4 \\ &= 1896 + 120 \\ &= 2016 \end{aligned}$$



∴ The 31st Olympic Games took place in 2016.

Remainder Problems

If we have a division statement, $29 \div 4 = 7 \text{ R}1$,

29	÷	4	=	7	R1
dividend		divisor		quotient	remainder

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

Problem

How many numbers, when divided by 6, have the same quotient and remainder?

The maximum remainder when divided by 6 is 5.

$$6 \times \text{Quotient} + \text{Remainder} = \text{Dividend}$$

$$6 \times 5 + 5 = 35$$

$$6 \times 4 + 4 = 28$$

$$6 \times 3 + 3 = 21$$

$$6 \times 2 + 2 = 14$$

$$6 \times 1 + 1 = 7$$

\therefore 5 numbers have the same quotient and remainder.